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The cyclotron resonance of a three-dimensional impurity magnetopolaron in the presence of a strong parabolic potential

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Abstract. In this investigation, we study the combined effects of a parabolic potential and a Coulomb impurity on the cyclotron resonance of a three-dimensional bound magnetopolaron, under the condition of strong parabolic potential. We only consider the case of weak electron–LO-phonon coupling and apply Larsen's perturbation method to the calculation of energy levels. The polaron resonances in a low-lying energy level are studied and certain cyclotron effective masses are calculated. We apply our calculations to GaAs.

1. Introduction

In recent years there have been such great advances in expitaxial techniques for the growth of semiconductor structures that it is possible to grow wire-like structures [1–8] on the low-nanometre scale. There have been many experimental and theoretical studies on the optical properties of these quasi-one-dimensional quantum systems [8–13]. Changes in the optical properties due to lateral confinement, such as blue shift of the luminescence and splitting of the subbands, were studied [11–13]. Also, a drastic change of the optical polarization selection rules and a large red shift of the photoluminescence and photoluminescence excitation spectra were observed [8]. There have also been extensive studies on the electronic properties of quantum-well wires (QWWs): for example, the impurity-limited mobility [14, 15]; the binding energy of hydrogenic impurity states in QWWs with finite or infinite square-well potential [16, 17]; the energy shifts and the effective mass of an electron and the exciton binding energies associated with the effects of the electron–LO-phonon interaction [18, 19]; the mobility of electrons scattered by impurities and by acoustic and polar optical phonons [20]; and the scattering of electrons by polar optical phonons [21, 22].

In most of the above-mentioned theoretical works concerning the quantum wire, the electron is assumed to be confined within a square-well potential. This is obviously not the realistic case since the potential of the electrons (apart from the Coulomb potential due to impurities) within the wire should not be zero. Indeed, Kash *et al* [23] have recently observed good evidence for the existence of a parabolic potential well in a quantum wire produced by strain gradients using a patterned carbon stressor, so it is significant to study the effects of the parabolic potential on the electronic properties in a quantum wire. In fact, Yildirim and Ercelebi [24, 25] have theoretically achieved the effective reduction of

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dimensionality for free polaron located initially in a three-dimensional system, and Li *et al* [26] studied the effects of the parabolic potential and confined phonons on a free polaron in a quantum wire.

The impurity-polaron problem in the bulk three-dimensional case has been studied using the weak-coupling perturbative approach [27, 28], variational methods [29–31], and the path integral technique [32]. The cyclotron resonance of the three-dimensional magnetopolaron and the impurity magnetopolaron in some III–V compounds has attracted intensive theoretical and experimental research interests [33–41]; the pinning effect and the resonant polaron effect were studied in great detail. Also, there have been many experimental [42–48] and theoretical [49–56] investigations on the magnetopolaron and the impurity magnetopolaron in two-dimensional and quasi-two-dimensional electronic systems, studying the influence of lower dimensionality on the resonant polaron effect. For a QWW, Vasilopoulos *et al* [57] have studied the magnetophonon resonances. They found that the cyclotron resonance frequency decreases with increasing size of the quantum wire. Also, Zhou *et al* [58] studied the cyclotron resonance of magnetopolarons in QWWs surrounded by a vacuum; they discovered that the absolute value of the electron–phonon interaction energy and the cyclotron resonance frequency decrease with increasing size of the QWWs.

In this work, we investigate the combined effects of a parabolic potential and a Coulomb impurity on the cyclotron resonance of a three-dimensional bound magnetopolaron. We will only consider the case of strong parabolic potential ($\omega \geqslant 4\omega_{LO}$) and weak electron–LO-phonon coupling. Larsen's perturbation method [33, 41] will be used and the Fröhlich Hamiltonian will be treated as a perturbation. We study the polaron resonances in a low-lying energy level and certain cyclotron effective masses are calculated. We then apply our results to the case of GaAs.

2. The Hamiltonian

Suppose that we have a Coulomb impurity located at the origin of a three-dimensional polar crystal, a constant magnetic field B pointing in the direction of the z-axis, and also a parabolic potential of frequency ω, which is circular with respect to the z-axis. Under the isotropic effective-mass approximation, the Hamiltonian of the system, which consists of an electron interacting with the bulk LO phonons in the polar crystal, can be written as

$$H = \frac{1}{2m}\left(P_x - \frac{eB}{2}y\right)^2 + \frac{1}{2m}\left(P_y + \frac{eB}{2}x\right)^2 + \frac{P_z^2}{2m} + \sum_q \hbar\omega_{LO} a_q^+ a_q - \frac{e^2}{4\pi\varepsilon_\infty r} + \frac{m\omega^2}{2}(x^2+y^2) + H_1 \tag{1}$$

where

$$H_1 = \sum_q (V_q e^{iq\cdot r} a_q + V_q^* e^{-iq\cdot r} a_q^+) \tag{2}$$

$$V_q = (i\hbar\omega_{LO}/|q|)(\hbar/2m\omega_{LO})^{1/4}(4\pi\alpha/V)^{1/2} \tag{3a}$$

and

$$\alpha = (e^2/8\pi\hbar\omega_{LO})(2m\omega_{LO}/\hbar)^{1/2}(1/\varepsilon_\infty - 1/\varepsilon_0). \tag{3b}$$

Here the electron has position vector $r = (x, y, z)$ and momentum $P = (P_x, P_y, P_z)$. ω_{LO} is the bulk LO phonon frequency and $q = (q_x, q_y, q_z)$ is the bulk phonon wave vector. a_q and a_q^+ are the annihilation and creation operators of the bulk LO phonon, respectively. Also, m is the electron band mass and V is the volume of the crystal. Finally, ε_∞ and ε_0 are the optical and static dielectric constants, respectively.

3. The method of solutions

The total Hamiltonian H (1) represents a complicated system. We cannot find out the exact solutions even when the polaron effects are completely ignored. We would like to point out that there is a special case in which H can be solved by using a perturbative method, namely, the case where l/a_0 is small, where $l = (\hbar/m\omega_{\text{eff}})^{1/2}$ with $\omega_{\text{eff}} = \sqrt{\omega^2 + \frac{1}{4}\omega_c^2}$ is the effective radius of confinement of the electron in the xy-plane (see (7)), and $a_0 = 4\pi\varepsilon_\infty\hbar^2/me^2$ is the Bohr radius (which is also the radius of the ground state of the one-dimensional hydrogen problem $P_z^2/2m - e^2/4\pi\varepsilon_\infty|z|$). Under the condition that l/a_0 is small, the motion of the electron in the xy-plane is strongly confined to the origin. Hence $\sqrt{x^2+y^2} \ll |z|$ or

$$1/r = 1/\sqrt{x^2+y^2+z^2} \simeq 1/|z|.$$

In view of this approximation, we will write the Coulomb term as

$$-e^2/4\pi\varepsilon_\infty r = -e^2/4\pi\varepsilon_\infty|z| + (-e^2/4\pi\varepsilon_\infty r + e^2/4\pi\varepsilon_\infty|z|) \tag{4}$$

and treat the second term on the right-hand side of (4) as a small perturbation.

l/a_0 can be expressed as

$$l/a_0 = (r_p/a_0)\sqrt{2}/[(\omega/\omega_{LO})^2 + \tfrac{1}{4}(\omega_c/\omega_{LO})^2]^{1/4}$$

where $r_p = (\hbar/2m\omega_{LO})^{1/2}$ is the polaron radius. In the case of GaAs ($\varepsilon_\infty = 10.9$, $\hbar\omega_{LO} = 36.63$ meV, and $m = 0.066m_e$ (m_e is the electron bare mass)), we have $r_p/a_0 = 0.454$. Since $B = 40$ T is practically a very high magnetic field, we will only consider the range $0 \leqslant B \leqslant 40$ T, or $0 \leqslant \omega_c/\omega_{LO} \leqslant 2$. The condition of $l/a_0 \leqslant 0.32$ or $(l/a_0)^2 \leqslant 0.1$ is sufficient for the above-mentioned perturbative treatment of the Coulomb term. For any value of B between 0 T and 40 T, we use the above explicit formula for l/a_0 to determine the minimum value of the parabolic frequency ω that makes $l/a_0 \leqslant 0.32$. We find that $\omega = 4.03\omega_{LO}$ for $B = 0$ T, and $\omega = 3.9\omega_{LO}$ for $B = 40$ T. Hence we will study the cyclotron resonance problem in the region $0 \leqslant B \leqslant 40$ T and $\omega \geqslant 4\omega_{LO}$. Also, we will only consider the case of weak electron–LO-phonon coupling, where Larsen's perturbation method is applicable.

Correspondingly, we rewrite the total Hamiltonian H as follows:

$$H = \tilde{H} + \sum_q \hbar\omega_{LO} a_q^+ a_q + H_I \tag{1a}$$

where

$$\tilde{H} = H_0 + \Delta H \tag{5a}$$

$$H_0 = (1/2m)(P_x - (eB/2)y)^2 + (1/2m)(P_y + (eB/2)x)^2 + (m\omega^2/2)(x^2+y^2) + P_z^2/2m - e^2/4\pi\varepsilon_\infty|z| \tag{5b}$$

$$\Delta H = -e^2/4\pi\varepsilon_\infty r + e^2/4\pi\varepsilon_\infty|z|. \tag{5c}$$

The condition $l/a_0 \leqslant 0.32$ means that $l \leqslant 0.32a_0 = 2.8$ nm, or that the wire diameter ($2l$) is less than 5.6 nm. This dimension is more than an order of magnitude smaller than current technology allows. Also, present quantization energies lie in the range $\hbar\omega = 1$–10 (at most) meV in GaAs; the strong parabolic potential condition $\omega \geqslant 4\omega_{LO}$ means $\hbar\omega \simeq 150$ meV.

To summarize, ΔH will be treated as a perturbation to H_0, and we will in turn treat H_1 as a perturbation to $\tilde{H} + \sum_q \hbar\omega_{LO} a_q^+ a_q$. We will adopt Larsen's approach and use the Wigner–Brillouin perturbation theory (WBPT) in the case of the resonant polaron.

The energy levels and states of H_0 are given below [59, 60]:

$$E^{(0)}_{n,m,k} = (2n + |m| + 1)\hbar\omega_{\text{eff}} + \tfrac{1}{2}m\hbar\omega_c - (1/k^2)R_y \tag{6a}$$

$$\Phi^{(0)}_{n,m,k} = \Psi_{n,m}(x, y)\phi_k(z) \tag{6b}$$

$$\tilde{\Phi}^{(0)}_{n,m,k} = \Psi_{n,m}(x, y)\tilde{\phi}_k(z) \tag{6c}$$

$$n = 0, 1, 2, \ldots \qquad m = 0, \pm1, \pm2, \ldots \qquad k = 1, 2, 3, \ldots$$

where

$$\Psi_{n,m}(x, y) = \left(1/\sqrt{\pi}\right)(l)^{-(|m|+1)}\sqrt{n!/[(m+n)!]^3}\,\rho^{|m|}e^{-\rho^2/2l^2}L^{|m|}_{|m|+n}(\rho^2/l^2)e^{im\theta} \tag{7}$$

$$\phi_k(z) = (-1/k!)\sqrt{2/k^5a_0^3}|z|e^{-|z|/ka_0}L^1_k(2|z|/ka_0) \tag{8a}$$

$$\tilde{\phi}_k(z) = \begin{cases} \phi_k(z) & z > 0 \\ -\phi_k(-z) & z < 0 \end{cases}. \tag{8b}$$

Here n is the Landau quantum number and m is the z angular-momentum quantum number. $\phi_k(z)$ and $\tilde{\phi}_k(z)$ are the energy states of the one-dimensional hydrogen problem $P_z^2/2m - e^2/4\pi\varepsilon_\infty|z|$, with even and odd parity, respectively. $L^{|m|}_{|m|+n}$ and L^1_k are the associated Laguerre polynomials. $R_y = me^4/32\pi^2\varepsilon_\infty^2\hbar^2$ is the Rydberg constant, $\omega_c = eB/m$ is the cyclotron frequency, $\omega_{\text{eff}} = \sqrt{\omega^2 + \frac{1}{4}\omega_c^2}$, and $l = (\hbar/m\omega_{\text{eff}})^{1/2}$ is the effective radius of confinement in the xy-plane. Finally, $\rho = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$.

Let $\tilde{E}_{n,m,k}$ be the energy levels of $\tilde{H}$. We have, to the first order of perturbation,

$$\tilde{E}_{n,m,k} = E^{(0)}_{n,m,k} + \langle\Phi^{(0)}_{n,m,k}|\Delta H|\Phi^{(0)}_{n,m,k}\rangle. \tag{9}$$

Next, let $\tilde{E}_{n,m,k,i}$ be the energy levels of $\tilde{H} + \sum_q \hbar\omega_{LO} a_q^+ a_q$, where the index i is equal to the number of phonons. We have

$$\tilde{E}_{n,m,k,i} = \tilde{E}_{n,m,k} + i\hbar\omega_{LO}. \tag{10}$$

We will only consider the case of zero temperature, so we will restrict our discussions to the case of zero number of phonons, that is, $i = 0$. Let $\delta E^{(\text{el-ph})}_{n,m,k,0}$ be the perturbation on $\tilde{E}_{n,m,k,0}$ due to the Fröhlich Hamiltonian H_1. We have, to the second order of Rayleigh–Schrödinger perturbation theory (RSPT),

$$\delta E^{(\text{el-ph})}_{n,m,k,0} = -\sum_{n_1,m_1,k_1,q} |\langle\Phi^{(0)}_{n_1,m_1,k_1}|\langle q|H_1|0_q\rangle|\Phi^{(0)}_{n,m,k}\rangle|^2/(\hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1} - \tilde{E}_{n,m,k})$$

$$
\begin{aligned}
&- \sum_{n_1,m_1,k_1,q} |\langle \tilde{\Phi}^{(0)}_{n_1,m_1,k_1}|\langle q|H_I|0_q\rangle|\Phi^{(0)}_{n,m,k}\rangle|^2/(\hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1} - \tilde{E}_{n,m,k}) \\
&= -\alpha\hbar^2\omega_{LO}^2 r_p \frac{4\pi}{V} \\
&\times \sum_{n_1,m_1,k_1,q} \frac{(1/|q|^2)|\langle\Psi_{n,m}(x,y)|e^{iq_{11}\cdot\rho}|\Psi_{n_1,m_1}(x,y)\rangle|^2|\langle\phi_k(z)|e^{iq_z z}|\phi_{k_1}(z)\rangle|^2}{(\hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1} - \tilde{E}_{n,m,k})} \\
&- \alpha\hbar^2\omega_{LO}^2 r_p \frac{4\pi}{V} \\
&\times \sum_{n_1,m_1,k_1,q} \frac{(1/|q|^2)|\langle\Psi_{n,m}(x,y)|e^{iq_{11}\cdot\rho}|\Psi_{n_1,m_1}(x,y)\rangle|^2|\langle\phi_k(z)|e^{iq_z z}|\tilde{\phi}_{k_1}(z)\rangle|^2}{(\hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1} - \tilde{E}_{n,m,k})}
\end{aligned}
\tag{11}
$$

where $\rho = (x, y)$ and $q_{11} = (q_x, q_y)$. The perturbed energy levels are, up to the second order of perturbations,

$$E_{n,m,k,0} = \tilde{E}_{n,m,k} + \delta E^{(el-ph)}_{n,m,k,0}. \tag{12}$$

The condition of polaron resonance is, from equation (11), determined by

$$\tilde{E}_{n,m,k} = \tilde{E}_{n_1,m_1,k_1} + \hbar\omega_{LO} \tag{13}$$

for certain sets of n_1, m_1, and k_1. We will use the WBPT in the case of the resonant polaron. Thus, we solve the following equation for $E_{n,m,k,0}$:

$$
\begin{aligned}
E_{n,m,k,0} &= \tilde{E}_{n,m,k} - \alpha\hbar^2\omega_{LO}^2 r_p \frac{4\pi}{V} \\
&\times \sum_{n_1,m_1,k_1,q} \frac{(1/|q|^2)|\langle\Psi_{n,m}(x,y)|e^{iq_{11}\cdot\rho}|\Psi_{n_1,m_1}(x,y)\rangle|^2|\langle\phi_k(z)|e^{iq_z z}|\phi_{k_1}(z)\rangle|^2}{(\hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1} - E_{n,m,k,0})} \\
&- \alpha\hbar^2\omega_{LO}^2 r_p \frac{4\pi}{V} \\
&\times \sum_{n_1,m_1,k_1,q} \frac{(1/|q|^2)|\langle\Psi_{n,m}(x,y)|e^{iq_{11}\cdot\rho}|\Psi_{n_1,m_1}(x,y)\rangle|^2|\langle\phi_k(z)|e^{iq_z z}|\tilde{\phi}_{k_1}(z)\rangle|^2}{(\hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1} - E_{n,m,k,0})}.
\end{aligned}
\tag{12a}
$$

The numerical method will be used to solve (12*a*). We plot a straight line of slope of 45° representing the left-hand side of (12*a*), and a curve representing the right-hand side of (12*a*). The intersections of the straight line and the curve are the solutions we want.

4. Results and discussion

We will consider the following three cyclotron resonance frequencies:

$$\omega_c^*(0, -1, 1) = (E_{0,-1,1,0} - E_{0,0,1,0})/\hbar$$

$$\omega_c^*(0, 1, 1) = (E_{0,1,1,0} - E_{0,0,1,0})/\hbar$$

and

$$\omega_c^*(0, 0, 2) = (E_{0,0,2,0} - E_{0,0,1,0})/\hbar. \tag{14}$$

The corresponding cyclotron effective masses are respectively

$$m^*(0, -1, 1) = m\hbar\omega_c(E_{0,-1,1,0} - E_{0,0,1,0})^{-1}$$

$$m^*(0, 1, 1) = m\hbar\omega_c(E_{0,1,1,0} - E_{0,0,1,0})^{-1}$$

and

$$m^*(0, 0, 2) = m\hbar\omega_c(E_{0,0,2,0} - E_{0,0,1,0})^{-1} \tag{15}$$

where m is the electron band mass.

The condition of polaron resonance is determined by

$$\tilde{E}_{n,m,k} = \hbar\omega_{LO} + \tilde{E}_{n_1,m_1,k_1}. \tag{13}$$

The polaron-resonance condition (13) is different from the polaron-resonance condition of the well investigated case of no Coulomb impurity and no parabolic potential. From the formulae (6*a*) and (9), we see that the (resonant) values of the cyclotron frequency ω_c, at which (13) is satisfied, are dependent on the parabolic potential and Coulomb impurity. In the region $0\ T \leqslant B \leqslant 40\ T$, the lowest energy level in which polaron resonance occurs is $E_{0,1,1,0}$. Putting $(n, m, k) = (0, 1, 1)$ in and solving numerically equation (13), we find that numerous energy levels intersect $\tilde{E}_{0,1,1} - \hbar\omega_{LO}$ at various values of ω_c, specifically, those energy levels $\tilde{E}_{0,-1,k}$, where $k = 1, 2, 3, \ldots$. Since the index k is the quantum number of the one-dimensional hydrogen problem, as can be seen from equation (6*a*), we conclude that this complicated behaviour of the polaron resonance is due to the presence of the Coulomb impurity. On the other hand, when there is no Coulomb impurity or parabolic potential, it is well known [33, 41] that the lowest energy level in which the polaron resonance occurs is $n = 1$, where n is the Landau quantum number, and the resonant value of the cyclotron frequency is

$$\omega_c = \omega_{LO}. \tag{16}$$

For simplicity, we will only consider the polaron resonances due to the intersection of $\tilde{E}_{0,1,1}$ and $\tilde{E}_{0,-1,1} + \hbar\omega_{LO}$, and of $\tilde{E}_{0,1,1}$ and $\tilde{E}_{0,-1,2} + \hbar\omega_{LO}$ (i.e. $k = 1, 2$). For illustration, we apply our calculations to the case of GaAs, where $\alpha = 0.067$ (weak electron–phonon coupling). The numerical results are shown in figures 1–4.

In figure 1(*a*) and (*b*), the energy level $E_{0,1,1,0}$ is plotted against the cyclotron frequency ω_c, with two different values of the parabolic frequency, namely, $\omega = 4\omega_{LO}$ and $\omega = 5\omega_{LO}$, respectively. In both of (*a*) and (*b*), the dashed lines represent the unperturbed energy levels $\tilde{E}_{0,1,1}$, $\tilde{E}_{0,-1,1} + \hbar\omega_{LO}$ and $\tilde{E}_{0,-1,2} + \hbar\omega_{LO}$, and the solid lines and dotted lines represent the energy level $E_{0,1,1,0}$. For $\omega = 4\omega_{LO}$, the polaron resonances occur at $\omega_c = 1.000\omega_{LO}$ and $\omega_c = 1.083\omega_{LO}$ (see figure 1(*a*)). In the region either on the left-hand side of $\omega_c = 1.000\omega_{LO}$ or on the right-hand side of $\omega_c = 1.083\omega_{LO}$, the RSPT (11) and (12) is used to calculate $E_{0,1,1,0}$ and the result is represented by the two solid lines, while in the region around $\omega_c = 1.000\omega_{LO}$ and $\omega_c = 1.083\omega_{LO}$, we use the WBPT (12*a*) to calculate $E_{0,1,1,0}$ and the result is represented by three dotted lines, as is shown in figure 1(*a*). Next, for $\omega = 5\omega_{LO}$,

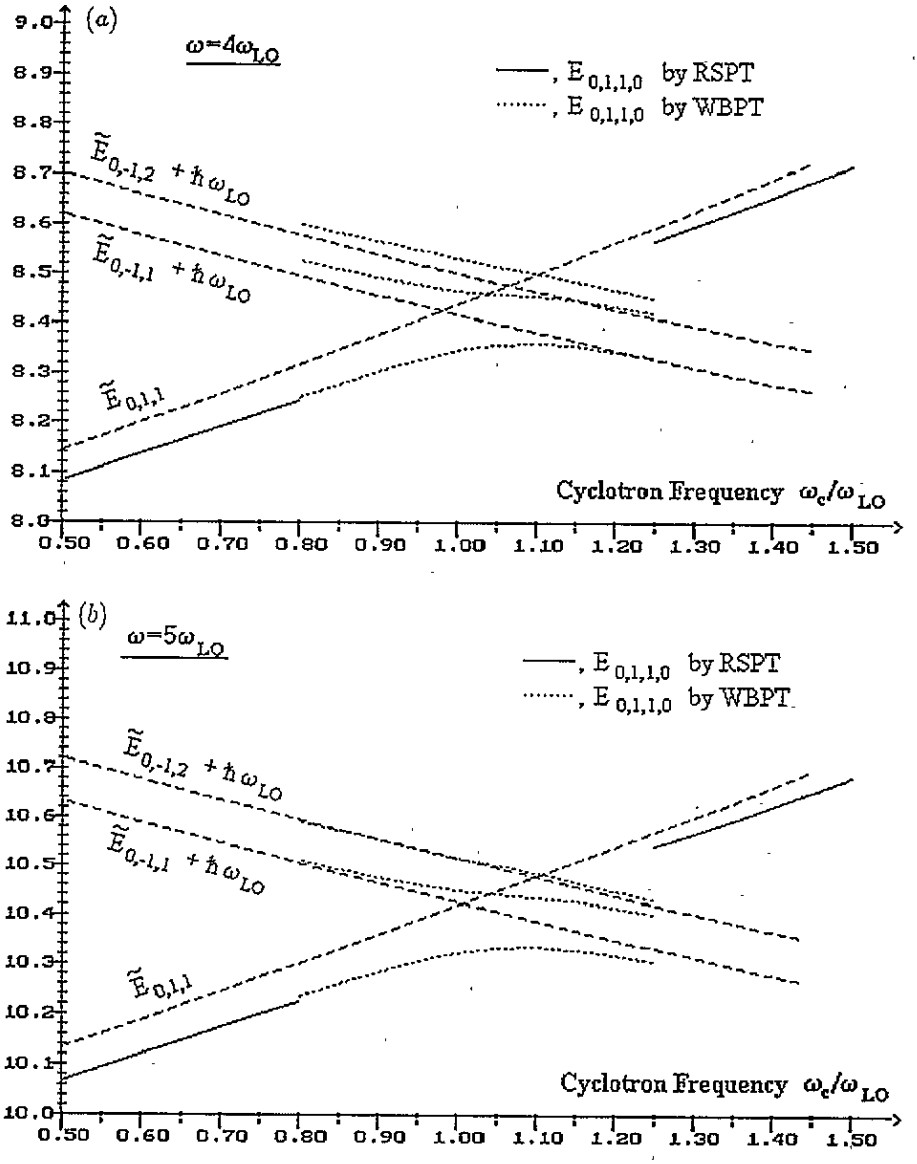

Figure 1. The cyclotron frequency dependence of the energy level $E_{0,1,1,0}$ of the three-dimensional bound magnetopolaron for the case of GaAs (*a*) at $\omega = 4\omega_{LO}$ and (*b*) at $\omega = 5\omega_{LO}$. The dashed lines represent $\tilde{E}_{0,1,1}$, $\tilde{E}_{0,-1,1} + \hbar\omega_{LO}$ and $\tilde{E}_{0,-1,2} + \hbar\omega_{LO}$; the solid lines represent $E_{0,1,1,0}$ by RSPT; the dotted lines represent $E_{0,1,1,0}$ by WBPT. All the energies are in units of $\hbar\omega_{LO}$.

the polaron resonances occur at $\omega_c = 1.000\omega_{LO}$ and $\omega_c = 1.0894\omega_{LO}$ (see figure 1(*b*)), and we use the RSPT and WBPT to calculate $E_{0,1,1,0}$ in a similar way as in the case of $\omega = 4\omega_{LO}$. We would like to point out that the pinning effect also exists in the presence of the Coulomb impurity and parabolic potential, and it is represented by the lowest dotted line in figure 1.

From the above discussions, we see that there is more than one resonant value of ω_c and the greater one is different from that of the case of no Coulomb impurity and no parabolic potential, where we have $\omega_c = \omega_{LO}$ (see (16)). We would like to re-iterate that the presence

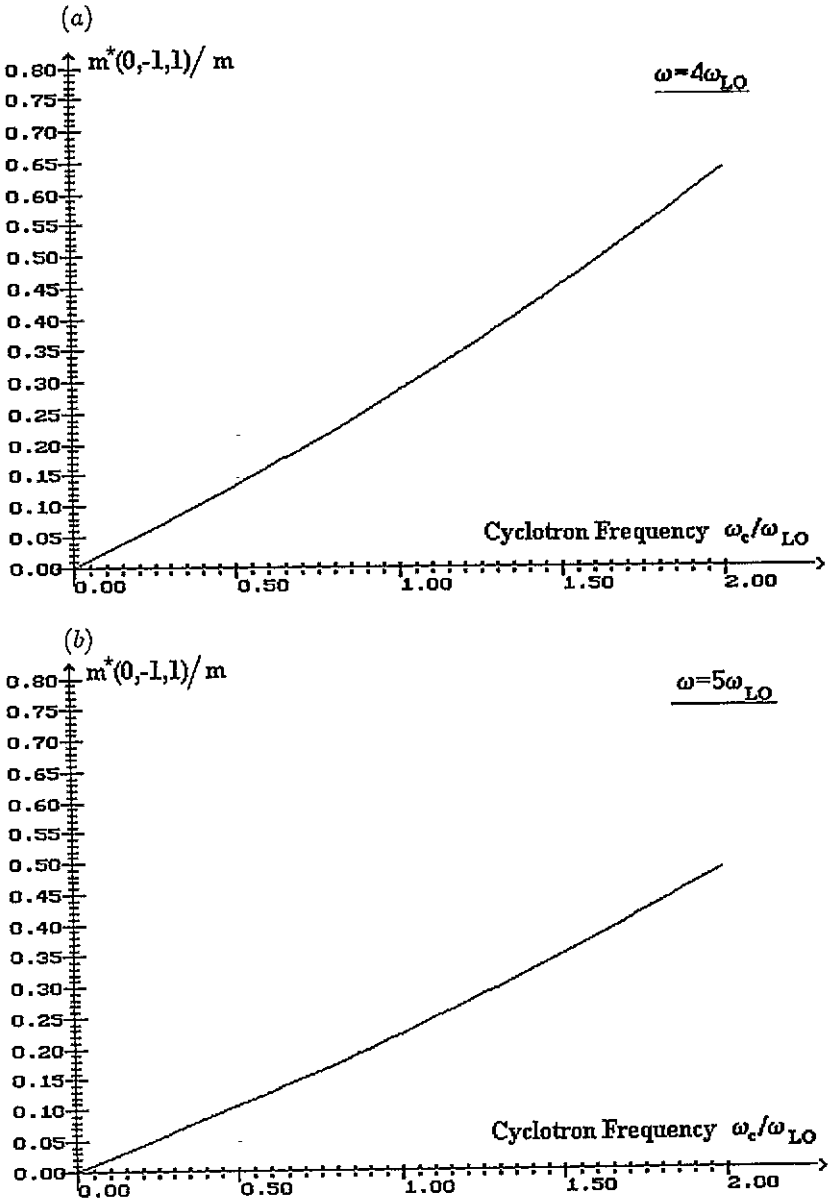

Figure 2. The cyclotron frequency dependence of the cyclotron effective mass $m^*(0,-1,1)$ of the three-dimensional bound magnetopolaron for the case of GaAs (a) at $\omega = 4\omega_{LO}$ and (b) at $\omega = 5\omega_{LO}$.

of the Coulomb impurity and parabolic potential change the resonant values of ω_c.

As mentioned above, we have for simplicity omitted the other polaron resonances due to the intersections of $\tilde{E}_{0,1,1} - \hbar\omega_{LO}$ and $\tilde{E}_{0,-1,k}$, where $k = 3, 4, \ldots$. We would like to point out that in principle if we take all these other polaron resonances into account and solve for $E_{0,1,1,0}$ using the WBPT (12a), we will obtain numerous (i.e. much more than three) solutions, leading to numerous dotted lines representing $E_{0,1,1,0}$ in both figure 1(a) and 1(b).

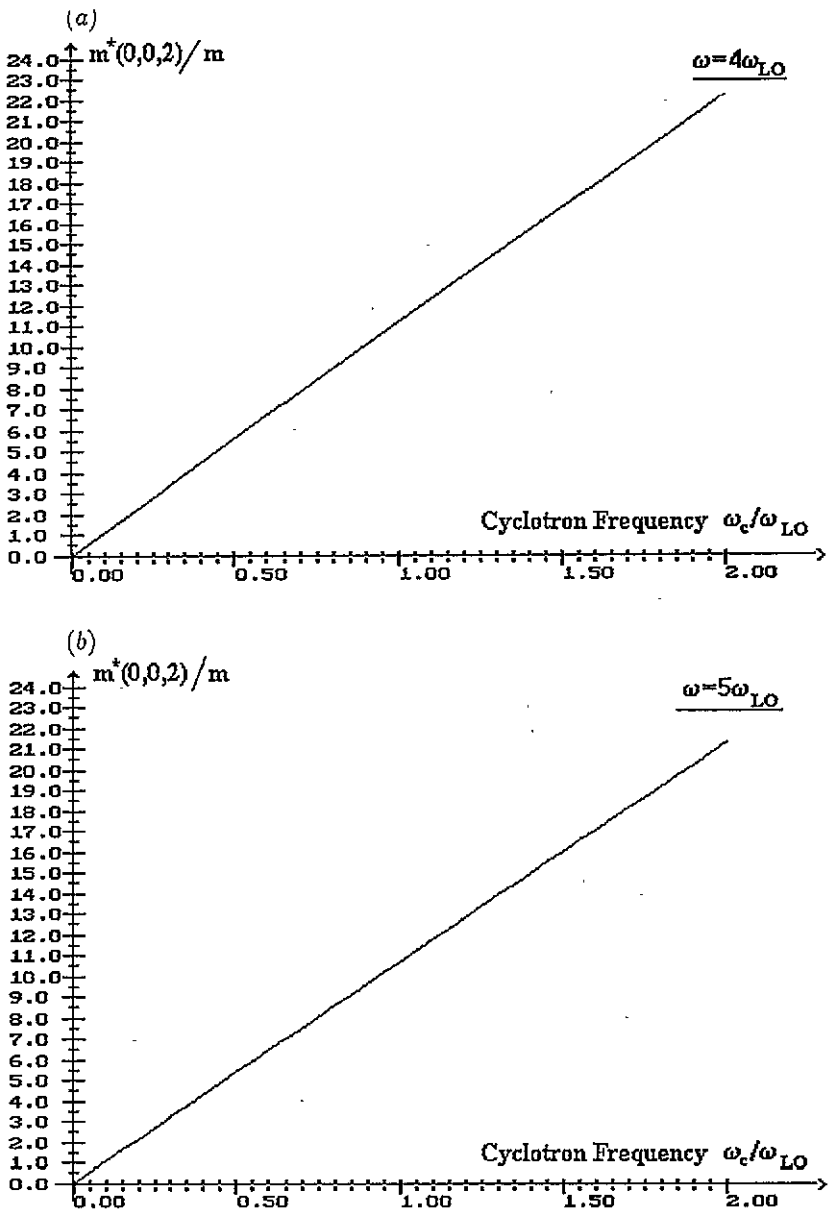

Figure 3. The cyclotron frequency dependence of the cyclotron effective mass $m^*(0, 0, 2)$ of the three-dimensional bound magnetopolaron for the case of GaAs (a) at $\omega = 4\omega_{LO}$ and (b) at $\omega = 5\omega_{LO}$.

We would like to stress again that this complicated behaviour of polaron resonance is due to the combined effects of the Coulomb impurity and electron–LO-phonon intersection.

In figures 2–4, the cyclotron effective masses given by (15) are plotted against ω_c for the case of GaAs, with $\omega = 4\omega_{LO}$ and $\omega = 5\omega_{LO}$. From figures 2 and 3, we see that both $m^*(0, -1, 1)$ and $m^*(0, 0, 2)$ increase with ω_c for a fixed value of ω, and figure 4 shows that $m^*(0, 1, 1)$ is multi-valued for each value of ω_c in the region around the polaron

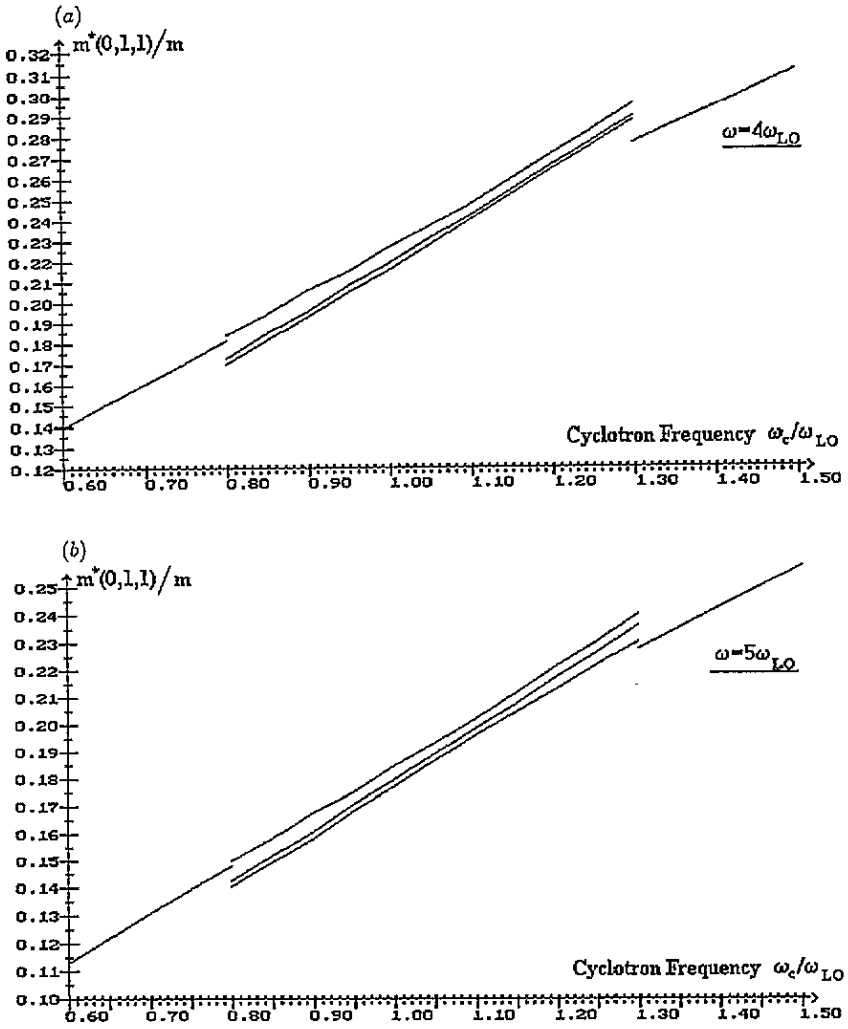

Figure 4. The cyclotron frequency dependence of the cyclotron effective mass $m^*(0, 1, 1)$ of the three-dimensional bound magnetopolaron for the case of GaAs (a) at $\omega = 4\omega_{LO}$ and (b) at $\omega = 5\omega_{LO}$.

resonances. This is due to the fact that $E_{0,1,1,0}$ behaves in a complicated way in the case of the resonant polaron, as is discussed above.

5. Conclusion

In this investigation, we studied the combined effects of the parabolic potential and Coulomb impurity on the properties of the cyclotron resonance of a bound magnetopolaron in polar crystals of weak electron–LO-phonon coupling, under the condition of strong parabolic potential ($\omega \geqslant 4\omega_{LO}$). Under the above condition, we adopted the decomposition (4) of the Coulomb term and treat the term ΔH and the Fröhlich Hamiltonian as perturbations. We

followed Larson's perturbative approach to calculate some low-lying energy levels and the corresponding cyclotron effective masses.

We applied our calculations to GaAs. We found that for each $\omega \geqslant 4\omega_{LO}$, the lowest energy level in which polaron resonance occurs is $E_{0,1,1,0}$, and it is multi-valued for each value of ω_c around the resonant values of ω_c. We concluded that the energy level $E_{0,1,1,0}$ splits into numerous sub-levels under the combined effects of the Coulomb impurity and electron–LO-phonon interaction. We also found that the 'pinning effect' exists in the presence of the Coulomb impurity and parabolic potential. Further, some of the (resonant) values of the cyclotron frequency ω_c (at which the resonance condition (13) is satisfied) were found to be different from the case of no Coulomb impurity and no parabolic potential, and they are dependent on the parabolic potential frequency ω. Finally, the cyclotron effective masses corresponding to some low-lying energy levels were calculated and we found that for fixed ω they increase with ω_c, with a remarkable feature in the polaron resonance in that they are multi-valued.

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